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Bernoulli process with 282 ky periodicity is detected in the R-N reversals of the earth's magnetic field

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ABSTRACT

Investigating the polarity time scales for the last 118 My a Bernoulli process with $p=\frac{1}{2}$ overriding with a Gaussian noise have been detected for the R-N magnetic reversals. Using the detected pattern the probability of an upcoming R-N reversal can be calculated between any time intervals. The Bernoulli trials are separated by 282 ky. This frequency is consistent with planetary and Milankovich cycles. The $p=\frac{1}{2}$ probability of the Bernoulli process could be the result of the positive and negative interferences of these cycles.

Key words: earth's magnetic field; R-N reversals; periodicity; probability

1. Introduction

Previous studies assumed that the mechanism responsible for the triggering of the magnetic reversal is the same in both cases. These studies were looking for sequences in the time series of the earth's magnetic field reversals by investigating the elapsed time between two consecutive reversals (e.g. Cox *et al.* 1981; Marzocchi and Mulargia, 1992; Pelletier, 1997; Consolini and De Michelis, 2003). The time sequence elapsed between the same polarity reversals has never been investigated.

It has been proposed that the two kinds of magnetic reversals, normal to reverse (N-R), and reverse to normal (R-N) might be triggered by different mechanisms (Garai, 2001). If the two different kinds of magnetic reversals are triggered by different mechanisms then by looking for a pattern in the sequences of the reversals would more accurately be detected by focusing analysis on those reversals, which are triggered by a common mechanism. In the present study the periodicity of the R-N reversals will be investigated.

2. Data Analysis

Using the polarity time scales for the last 118 My (Cande, and Kent, 1995) the time elapsed between two consecutive R-N reversals $\Delta t_{(R-N)}$ has been determined first.

$$\Delta t_{(R-N)} = t_{(R-N)_{i+1}} - t_{(R-N)_i} \tag{1}$$

where $t_{(R-N)_i}$ is the time before present of the ith R-N magnetic reversal. The number of data for longer time periods ($\Delta t_{(R-N)}$>1.6 My.) is very few. These long periods were omitted because the available data is insufficient for statistical analysis. The frequency distribution of Δt_{R-N}

reveals five relatively distinct groups (Fig. 1). If one assumes a Gaussian distribution for each of the groups, then these distributions overlap and the analyses are significantly more complicated. However, since the means of the various groups lay several standard deviations away from each other, a more tractable way to model the groups is to determine a point between them such that the probability of a point being miscategorized is minimized. To make this precise, consider G_1 and G_2 data set for two consecutive groups which have (mean, standard deviation) (μ_1, σ_1) and (μ_2, σ_2) respectively, and $\mu_1 < \mu_2$. Then we want to find $\mu_1 < s_{1\text{-}2} < \mu_2$ such that $p_1(X \geq s_{1-2}) + p_2(X < s_{1-2})$ is minimized, where X is a variable equal to Δt_{R-N}, p_1 is a probability calculated from a Gaussian density function with parameters of μ_1, σ_1, and p_2 is a probability calculated from a Gaussian density function with parameters of μ_2, σ_2. At first the groups were separated by visual inspection and parameters (μ_i, σ_i) were calculated for each group (G_i). Using the calculated group parameters the minimum points $s_{i-(i+1)}$ between each consecutive group were determined. If this minimum point coincided with the initial assumption then the group separation was accepted. If the minimum point did not coincide with the initial separation then the above procedure was repeated using the new minimum point for the separation. Calculations were repeated as long as all the calculated $s_{i-(i+1)}$ coincided with the initial separation point. The calculated parameters of the separated groups are given in table 1. The means of the overlapping Gaussian distributions are 3.86σ apart, therefore, the overlapping data on each side of the distribution is less than 1.5 percent. This small percentage of overlapping does not have significant affect on the statistical parameters. Additionally, the cut off parts for groups 2, 3, and 4 are close to symmetrical further reducing the overlapping affect.

Analyzing the calculated statistical parameters, the mean values and standard deviations of these separated groups it was found that the mean values are integer multiples of the smallest mean, revealing a periodic pattern. If we let the normalize mean of a group denote the mean of that group divided by its group number (e.g. group 5's normalized mean is 0.28376 My), then we find that the normalized means range from 0.280 to 0.287 My. The weighted average of the means is 0.28214 My, the corresponding standard deviation is 0.073 My

The frequency distribution of the reversal time shows that the R-N reversal is more likely to have a shorter term than a longer one. Calculating the relative frequency that a reversal falls into a group it was find that the frequency for the first group with mean of 0.28 My is 0.5. The frequency of the second group (mean 0.56 My) is 0.21, while the frequency for the third group (mean 0.85 My) is 0.12 etc. [Tab. 1]. This is just a Bernoulli process with $p=\frac{1}{2}$ overriding with a Gaussian noise. This means that for every ~0.28 My, elapsed since the previous R-N reversal, there is approximately 50% chance that the earth's magnetic field would make a shift from a reversed to a normal orientation (Fig. 2).

3. Calculating the probability of an R-N Reversal

The detected pattern and the statistical parameters allow one to predict the probability of the next R-N reversal between any time intervals. The probability of a Bernoulli trial is.

$$P\ (n) = p^{n}(1-p)^{1-n} \quad \text{for} \quad n = 0,\ 1 \tag{2}$$

where $p = \frac{1}{2}$. If zero of n means no R-N reversal, while one is equivalent with an R-N reversal then the probability of the ith R-N magnetic reversal is:

$$P\ (R-N)_i = p^n (1-p)^{1-n} = 0.5 \tag{3}$$

The uncertainty of the time of the Bernoulli trial is very high therefore the discrete Bernoulli trial should be replaced with a Gaussian distribution.

$$P\ (R-N)_i = p^n (1-p)^{1-n} \int_{-\infty}^{+\infty} P(t)_i \, dt = 0.5 \tag{4}$$

where $P(t)_i$ is the Gaussian density function for the ith trial.

$$P(t)_i = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t+780-it_0)^2}{2\sigma^2}} \tag{5}$$

where t is the time from present in ky, t_0 is the mean, and σ is the variance. The most recent R-N magnetic reversal, Matuyama-Brunhes, has been used as reference; therefore, 780 ky has been added to the current time. In order to calculate the probability of the occurrence of an R-N reversal between time t_1 and t_2 $[P(R-N)_{t_1-t_2}]$ first the sequence number of the Bernoulli trial (i) should be identified. The sequence numbers can be determined from equation 6 by keeping only the integer parts of the calculated values.

$$\frac{t_1 + 780}{t_0} + 0.5 \quad \text{and} \quad \frac{t_2 + 780}{t_0} + 0.5 \tag{6}$$

If the beginning and the end of the investigated period falls into the same Bernoulli trial ($i_1 = i_2$) then the probability of the event can be calculated by integrating equation 5 between the time t_1 and t_2.

$$P(R-N)_{t_1-t_2} = p^n (1-p)^{1-n} \int_{t_1}^{t_2} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(t+780-it_0)^2}{2\sigma^2}} dt \tag{7}$$

The parameters, mean 282 ky, and variance 73 ky, determined by this investigation should be used for the calculation. If the time period covers two or more consecutive Bernoulli trials ($i_1 \neq i_2$) then the probability of the R-N reversal can be calculated as:

$$P(R-N)_{t_1-t_2} = \{1-[p^n(1-p)^{1-n}]^{(z+1)}\}\left[0.5 - \frac{\int_{-\infty}^{t_1} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(t+780-i_1t_0)^2}{2\sigma^2}} dt + \int_{t_2}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(t+780-i_2t_0)^2}{2\sigma^2}} dt}{z+1}\right] \quad (8)$$

where $z = i_2 - i_1$.

4. Correlation with Astronomical Cycles

The presence of the planetary cycles in the strength and the inclination of the earth magnetic field for the past 2.25 has been detected by Yamazaki, and Oda (2002). The Bernoulli process of the R-N reversals of the earth's magnetic field has been stable at least for the time period of the data, which is about 100 My. With this time length only astronomical cycles are known to remain stable.

The frequency of the Bernoulli trials is a harmonic of the beat frequency (93,418.3 y.) of the giant planets (Shirley, and Fairbridge, 1997). The bit frequency of the global climate oscillation (Milankovich, 1930) is also consistent with the frequency of the Bernoulli trials.

Impact initiated global cooling has been proposed as a possible triggering mechanism for the magnetic reversals (Muller, and Morris, 1986). This model is consistent with geological observations for the last two R-N reversals (Glass, 1990). If impact initiated global cooling can

trigger an R-N reversal then the $p=\frac{1}{2}$ probability of the Bernoulli process could be explained as positive and negative interferences between the Milankovitch and planetary cycles.

5. Conclusion

In this study the time sequence elapsed between two consecutive R-N reversals has been analyzed for the last 118 My. The pattern consistent with a Bernoulli process with $p=\frac{1}{2}$ overriding with a Gaussian noise. The frequency of the detected Bernoulli trials is 282 ky. This frequency is consistent with planetary and global climate cycles. The $p=\frac{1}{2}$ probability of the Bernoulli process can be explained as positive and negative interferences between these cycles.

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Figure Captions

Fig. 1 The distribution of the time intervals elapsed between two consecutive R-N reversals, and the best fitting normal distributions of the separated groups.

Fig. 2 The probability distribution of the R-N reversals. The calculated statistical parameters contain the relatively high uncertainty of the geologic time determination (dashed lines). If the R-N reversals are generated by astronomical cycles then the uncertainty of the parameters should be significantly smaller (continuous lines).

Table 1.

Group Number	Time Range [My.]	Number Of Events	Frequency	Group Average [My.]	Normalized Average [My.]	Group Standard Deviation
1	0.079-0.436	40	0.5000	0.280	0.280	0.0886
2	0.465-0.710	17	0.2125	0.563	0.281	0.0621
3	0.768-0.862	10	0.1250	0.862	0.287	0.0537
4	1.069-1.232	7	0.0875	1.139	0.285	0.0504
5	1.350-1.520	6	0.0750	1.419	0.284	0.0578
Total:	**0.079-1.520**	**80**	**1.0000**	**Average:**	**0.282**	**0.0730**

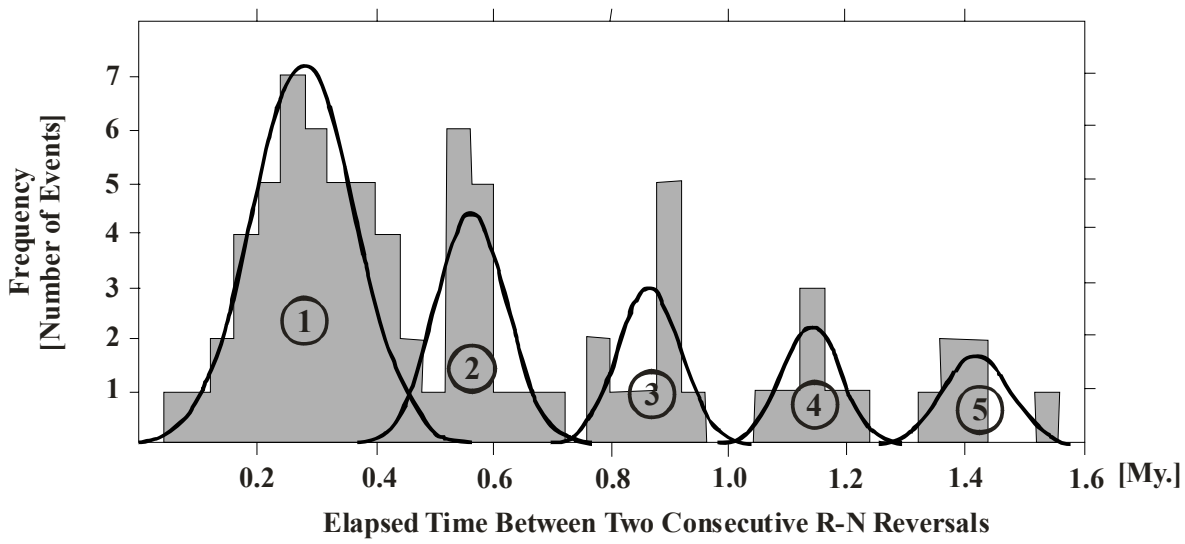
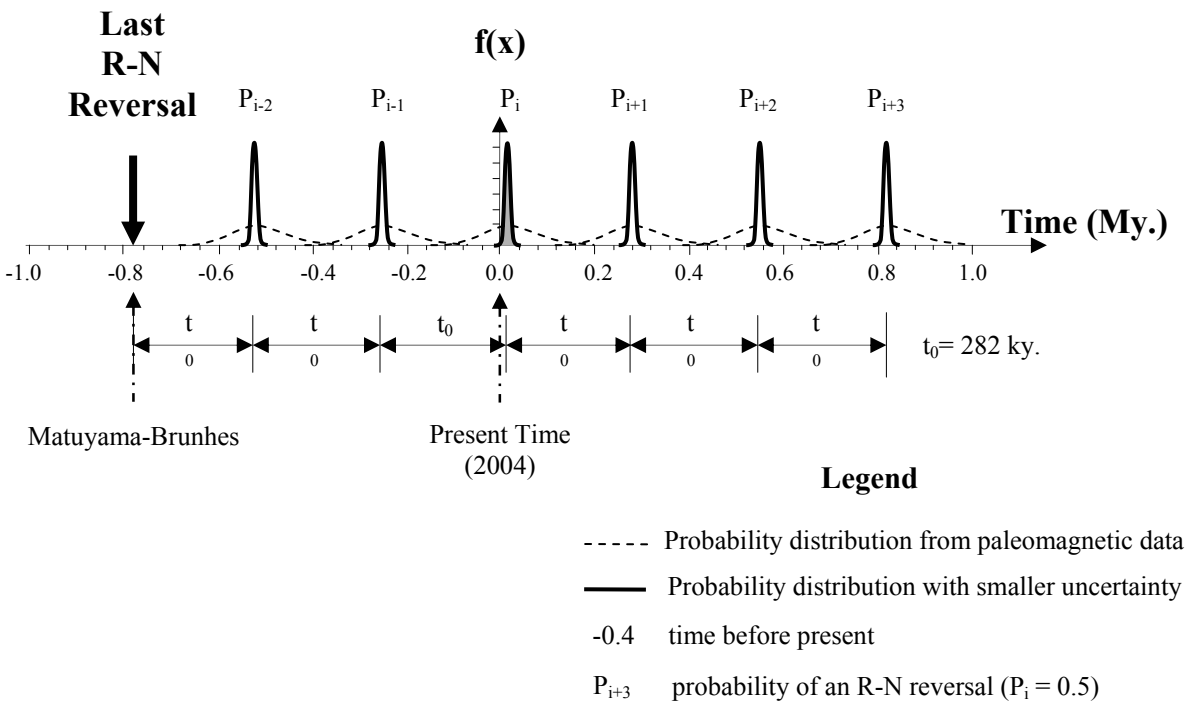

Frequency
[Number of Events]
7
6
5
4
3
2
1
1
2
3
4
5
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
[My.]
Elapsed Time Between Two Consecutive R-N Reversals

Figure 1.



Legend

- - - - - Probability distribution from paleomagnetic data

——— Probability distribution with smaller uncertainty

-0.4 time before present

P_{i+3} probability of an R-N reversal (P_i = 0.5)

Figure 2.