Regional modelling of Europian magnetic observatory data: search for the secular variation anomalies

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ABSTRACT

In a region without sources, magnetic field **B** can be represented as a gradiend of the scalar potential(Φ) which satisfies the Laplace equation. When the observations are available over the hole sphere, the problem can be solved by the method of spherical harmonic analyses (SHA) in terms of Legendre polynomials in colatitude and trigonometric functions in longitude. However, over the restricted area, these same Legendre polynomials and trigonometric functions are no longer the most appropriate basic functions for fitting the potential field model over such small area. The method for regional modelling of fields that can be expresed as the gradient of the scalar potential is known as spherical cap harmonic analyses (SCHA). By inverse problem one can derive a mathematical model, but without the regularization data will be overfit with the errors. The goal is to find the simple possible model that can fit the data within the estimated uncertainties.

Geomagnetic secular variation SV - the slow change of the Earth's main magnetic field is a topic of nowadays studies and disscusions because it's better understanding can help studing the core dynamic. The transient magnetic field will induce the secondary magnetic field in the electrically conducting materials of the Earth. The long standing question is whether there exist secular variation anomalies with the length scale less than 1000 km. This length scale is too small to originate from the core-mantle boundary so if those anomalies exist they are revealing electromagnetic induction anomalies originating in the upper mantle.

Geomagnetic observatories with good time-series of the data are dense in Europe and it should be possible to derive the regional model which fits the data well by using those data. I attempt to make such a model, the data will be simultaneously modelled in time and space. Applying the physical regularization the smoothest model required by the actual data will be carried out.

Data series will be compared with the sinthetic data obtained using Comprehensive model, phase3. To avoid the numerical instability at the edge of the choosen cap, since those regions are sparse of real data,the cap boundary regions will be filled with produced synthetic data.

Furthermore, I intend to compare the global model obtained with the CHAMP satellite with regional model for all field components and their secular variations.

1. INTRODUCTION

The Earth magnetic field is caused by sources in the Earth's core (internal part) and currents in the ionosphere and magnetosphere (external part). It changes in time on all observable time scales. The slow, secular variations (period of centuries) are due to the action of geodynamo, while more faster changes of the field (period between years and seconds) are caused by the external sources.

Geomagnetic secular variation, the slow change of the Earth's main magnetic field is a topic of nowadays studies and disscusions because it's better understanding can help studing the core dynamic. The transient magnetic field will induce the secondary magnetic field in the electrically conducting materials of the Earth. Variations with timescales shorter than a year or two are effectively screened by mantle conductivity and so are not detectable at the Earh's surface (Sabaka, 1997). The question that was addresed years ago (Rikitake, 1966) and is still present, is whether there exist secular variation anomalies with the length scale less than 1000 km. This length scale is too small to originate from the core-mantle boundary so if those anomalies exist they are revealing electromagnetic induction anomalies originating in the upper mantle.

In a region without sources, magnetic field **B** can be represented as a gradiend of the scalar potential(Φ) which satisfies the Laplace equation. When the observations are available over the hole sphere, the problem can be solved by the method of spherical harmonic analyses (SHA) in terms of Legendre polynomials in colatitude and trigonometric functions in longitude. However, over the restricted area, these same Legendre polynomials and trigonometric functions are no longer the most appropriate basic functions for fitting the potential field model over such small area. The method for regional modelling of fields that can be expresed as the gradient of the scalar potential is known as spherical cap harmonic analyses (SCHA).

At the geomagnetic observatories, the magnetic field is measured continuosly (what is imortant in order to separate internal from the external part) so they can provide a long time span of data. They are dense enough in Europe but not all over the globe. Also, in some countries the repeat stations measurements are carried out and with those data the more detailed studies of secular variations can be done and for practical purpose the geomagnetic charts can be updated. However, the problems with repeat station data are that they may contain errors of the order of magnitude of the secular variation itself. The raw data are momentary values measured at the survay stations on different days, they are generally reduced to "annual means" using recording of nearby observatory. Under the assumption that all geomagnetic variations are the same at the stations and the observatories, the data can be considered free of external field or no more influenced by the external variations than the observatories means. Induced variations can differ significantly over short distance, yielding the errors of 10 nT or more in the internal results. Furthermore, those data have to be reduced to common epochs what can distort the amplitude of the small scale secular variations.

In the attempt to make a good model of the secular variation of the geomagnetic field over the region of Europe, I suggest to use only the geomagnetic observatory data and not the repeat station data (at least in the first approach), since the observatory network is dense enough and the above mentioned problems when using repeat station data are avoided.

The simplest approach in secular variation modelling is to modell each field component separately with a spatial polynomial. As an alternative, Haines (1985) suggested Spherical cap harmonic analyses (SCHA) by which it is possible to model the full vectorial field as a negative potential of the field gradient.

By inverse problem one can derive a mathematical model, but without the regularization data will be overfit with the errors. This means that we can see small scale noise structure in the model, but which are caused by the data errors. The goal is to fit the data to the estimated uncertainty of the data, so to have the simplest possible model. There are two methods of the regularization: statistical regularization (Haines, 1985) in which coefficients that are considered as statistically insignificant are set to zero and physical regularization (Korte and Holme, 2003) which imploys the minimization of a certain features of the field over the cap surface.

Korte and Holme (2003) carefully investigated possible lithospheric secular variation anomalies with technique of SCHA applying physical regularization. They used data set of 30 years of European observatory measurements, repeat station and ground vector survays. Their data set was not good enough to conclude that secular variation anomalies exist, even if some of their model results suggests present of the small-scale features in the data. The problem of their data set were gaps in the data of some countries and they had to make reduction to the common epoch of 5 years.

I plan to study 53 years of the magnetic Europian observatory data with the main goal of searching for those kind of anomalies using the SCHA and physical regularization. The data are provided from 47 European observatorys and their locations are shown on Fig 1.

The first step will be in carefully checking the observatories time series for all field's components to assert possible jumps caused by changing of the locations of the observatories. Then, the individual epochs will be modelled separatelly. The appropriate half angle of the spherical cap should be found: observatorys should be placed well inside the cap, the numerical instability of the model near the cap boundary has to be reduced as much as possible and the model must behave well at lower harmonics. The latest will be stuided by inspection of the spatial power spectra of the SV field on different spherical caps.

I plan to add the synthetic data in the selected cap region where the data coverage is sparse, especially near the cap boundary in order to avoid the numerical instabilyties. For that purpose, the comprehensive model of the quiet time, phase 3, CM3 (Sabaka et al, 1991) will be used.

The smoothing conditions of the model will be chosen according to the data by considering the maximum degree of the polinomials, different damping norms and damping factors. The goal is to make the model smooth but not only by defining the truncation level of the series of the basic functions. Instead, I will try to trade of it against misfit of the model to the data.

Simultaneous modelling of the spatial and temporal distribution can give even better results (than modelling separatelly different epochs). The idea is to use the penalized least-square splines as developed for global modelling (Bloxham & Jackson, 1992; Constable & parker, 1988, 1991).

I hope to also have the opportunity to use the model prediction from CHAMP (CHAllenging Minisatellite Payload) satellite (property of GeoForshungsZentrum Potsdam, Germany) and make the regional model using this data. The obtained model can serve to test the regional model made before.

In section 2, I will explain the meodelling method. The method of Sherical harmonic analyses will be outlined and the regularization technique explained. In section 3, the idea for data modelling will be presented. The observatory data modelling, synthetic data modelling as well as the need of combining them will be discuss in the third section. The Comprehensive model (CM3) will be explained too.

There will be the short conclusion, section 4.

2. MODELLING METHOD

2.1. Spherical cap harmonic

For global modelling of the magnetic field the well-known technique is Spherical harmonic analyses (SHA). Haines (1985) developed a method to apply this technique on the limited region of the Earth and that method is called Spherical cap harmonic analyses (SCHA). The mathematical difference between global SHA and SCHA is in the basic functions. Those functions are completely orthogonal in SHA, but not in the SCHA.

In the source free region magnetic field $\mathbf{B}$ can be represent as the negative gradient of the scalar potential Φ, $\mathbf{B} = -\nabla\Phi$, where Φ has to satisfy the Laplace's equation: $\nabla^2\Phi = 0$. The general solution of the Laplace's equation in the case of SCHA is:

$$\Phi(r,\theta,\phi) = R_E \sum_{k=0}^{k_{\max}} \sum_{m=0}^{k} \left(\frac{R_E}{r}\right)^{n_k+1} [g_k^m \cos(m\phi) + h_k^m \sin(m\phi)] P_{n_k}^m(\cos\theta) \tag{1}$$

The potential is the function of radius r, colatitude θ and longitude ϕ. R_E is the mean radius of the Earth and $\{g_k^m, h_k^m\}$ are the Gauss coefficients. $P_{n_k}^m(\cos\theta)$ are not Legendre polynomials with integer degree n and order m. Instead, they are Legendre's functions with a non integer degree n_k which is the function of the colatitude of the spherical cap boundary. The order m is still integer

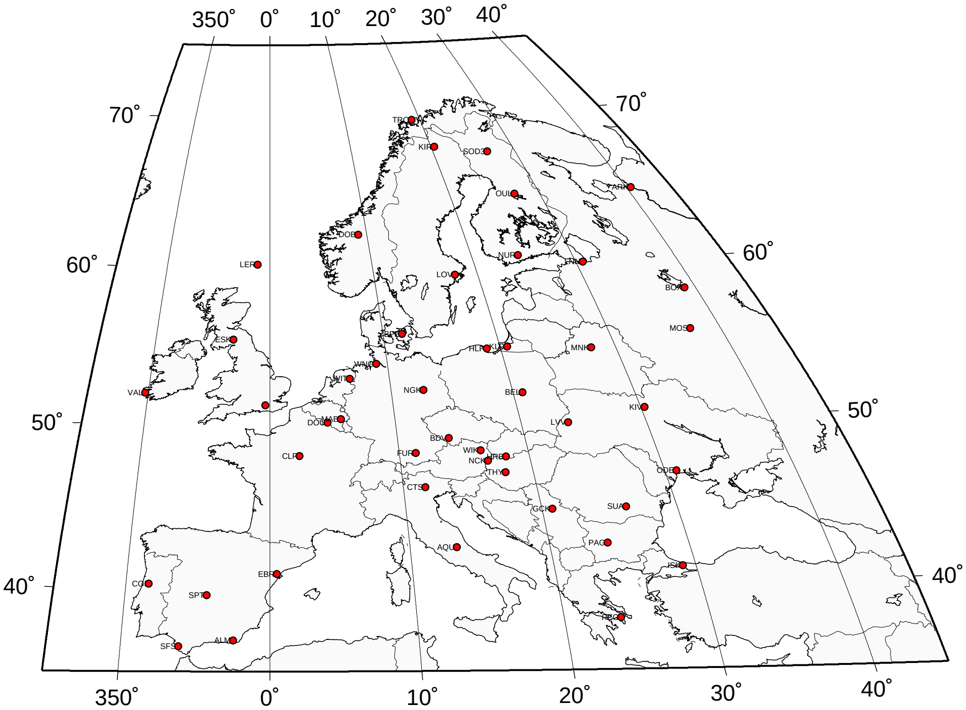

Fig. 1.— Network of mgeomagnetic observatory which will be used in the study of the secular variaton anomalies

as the potential must be continuous in ϕ. The boundary condition on θ, however, is only to be able to give an arbitrary function at the cap boundary. The values for $n_k(m)$ are found as the roots of the following equations (for given m, the different roots n are obtained):

$$\frac{\partial P_{n_k}^m}{\partial \theta} = 0 \tag{2}$$

or

$$P_{n_k}^m = 0 \tag{3}$$

The two sets of basis functions are denoted by the fact that for one set the difference $(k - m)$ is even, for the other $(k - m)$ is odd. All functions within one set are orthogonal, but the functions of one set are not completely orthogonal to those in the other. Haines also gives the equations for products of the functions:

$$\int_0^{\theta_0} P_{n_j}^m(\cos\theta)P_{n_k}^m(\cos\theta)\sin\theta d\theta = -\frac{\sin\theta_0}{[n_k-n_j][n_k+n_j+1]}P_{n_j}^m(\cos\theta_0)\frac{dP_{n_k}^m(\cos\theta_0)}{d\theta} \qquad (4)$$

for $(j-m)$ even and $(k-m)$ odd,

$$\int_0^{\theta_0}[P_{n_k}^m(\cos\theta)]^2\sin\theta d\theta = -\frac{\sin\theta_0}{2n_k+1}P_{n_k}^m(\cos\theta)\frac{\partial}{\partial n}\frac{dP_{n_k}^m(\cos\theta_0)}{d\theta} \qquad (5)$$

for $(k-m)$ even and

$$\int_0^{\theta_0}[P_{n_k}^m(\cos\theta)]^2\sin\theta d\theta = \frac{\sin\theta_0}{2n_k+1}\frac{dP_{n_k}^m(\cos\theta_0)}{d\theta}\frac{\partial}{\partial n}P_{n_k}^m(\cos\theta) \qquad (6)$$

for $(k-m)$ odd. Eqs (5) and (6) follow directly from eq (4), using L'Hôpital's rule in the limit that n_k tends to n_j.

2.2. Regularization technique

There are two methods of regularization: statistical and physical. In the statistical regularization (Haines, 1985) the coefficients that are considered as statistically insignificant are set to zero and physical regularization (Korte and Holme, 2003) which imploys the minimization of a certain features of the field over the cap surface. It was shown that statistical regularization lacks physical justification and that use of the individual coefficients are meaningful. The fact that in the SCHA the basic functions are not completely orthogonal leads to the egsistence of certain linear combinations of coefficients that significantly contribute to the field. For that reasons I will follow the physical regularization.

The mathematical model can be derived by linear inversion method. Without the regularization, the coefficients can be obtained by equation:

$$\gamma = \mathbf{Am} + e \qquad (7)$$

where γ is the data vector, $\mathbf{A}$ is the operator relating the data vector to the model, m is the model vector and e is the error vector. When use the regularization technique, the following function will be minimized:

$$(\gamma - \mathbf{Am})^T\mathbf{C}_e^{-1}(\gamma - \mathbf{Am}) + \lambda\mathbf{m}^T\mathbf{\Lambda m} \qquad (8)$$

where $(\gamma - \mathbf{Am})$ is the error vector given by the difference betwee data γ and the prediction of the model, $\mathbf{m}$ and $\mathbf{A}$ is the operator calculated from eq. (1) relating the data vector to the model.

$\mathbf{C}_e$ is the data error covariance matrix. The regularisation is given by the second term: $\mathbf{m}^T\mathbf{\Lambda}\mathbf{m}$ which is a quadratic norm of smoothness of the field over the spherical cap. $\mathbf{\Lambda}$ is a positive definite damping matrix. λ is a Lagrange multiplier. The maximum likelihood solution is

$$\hat{\mathbf{m}} = (\mathbf{A}^T\mathbf{C}_e^{-1}\mathbf{A} + \lambda\mathbf{\Lambda})^{-1}\mathbf{A}^T\mathbf{C}_e^{-1}\gamma \tag{9}$$

The damping matrix is determined by the norm.

In SHA, not only the basis functions but also the corresponding $\mathbf{B}_l^m$ are orthogonal over the sphere:

$$\int\limits_{\phi=0}^{2\pi}\int\limits_{\theta=0}^{\pi} \mathbf{B}_l^m \cdot \mathbf{B}_{l'}^{m'} \sin\theta d\theta d\phi = 0 \tag{10}$$

unless $l = l'$ and $m = m'$. Here, l has been used for the integer degree. The 2-norm of the mean field strength averaged over the spherical surface is given by

$$\langle \mathbf{B}\cdot\mathbf{B}\rangle = \sum_{l=1}^{\infty}(l+1)\left(\frac{R_E}{r}\right)^{2l+4}\sum_{m=0}^{\infty}((g_l^m)^2 + (h_l^m)^2) \tag{11}$$

so the damping matrix based on minimising this norm is diagonal with elements

$$f(l) = (l+1)\left(\frac{R_E}{r}\right)^{2l+4} \tag{12}$$

In SCHA,because of the incomplete orthogonality of the basis functions, the fields are orthogonal only to the same extent as the basis functions and the damping matrix is no longer diagonal. The functions are still ϕ-orthogonal, so integrals of products of functions with $m \neq m'$ are zero. As a result, most of the non-diagonal elements of the damping matrix are also zero, but the elements that combine odd and even harmonics of equal order m have finite values.

One possible regularisation is to minimise only the mean square radial component of $\mathbf{B}$. This norm is given by

$$\begin{aligned}\langle B_r^2\rangle &= \sum_{n_k}\sum_{n_j}\sum_{m}(g_{n_k}^m g_{n_j}^m + h_{n_k}^m h_{n_j}^m)(n_k+1)(n_j+1)\left(\frac{R_E}{r}\right)^{(n_k+n_j+4)} a \\ &\quad \cdot \int\limits_{\theta=0}^{\theta_0} P_{n_k}^m(\cos\theta)P_{n_j}^m(\cos\theta)\sin\theta d\theta\end{aligned} \tag{13}$$

where the θ-integral is given by eqs (4)-(6), The mean $\langle ...\rangle$ refers to the mean over the cap and the factor a is the result of the ϕ-integral, 2π for $m = 0$, π for $m \neq 0$, normalised for the area of the cap, $2\pi(1 - \cos\theta)$:

$$a = \begin{cases} 1/(1-\cos\theta_0) & : \quad m = 0 \\ 1/(2(1-\cos\theta_0)) & : \quad m \neq 0 \end{cases} \tag{14}$$

For $\theta_0 = \pi$ these values respectively become 0.5 and 0.25, which are the factors when normalising the ϕ-integral with the area of the whole sphere in the case of global spherical harmonics.

The square norm of the main field $\mathbf{B}$ is given by

$$\begin{aligned}\langle \mathbf{B} \cdot \mathbf{B} \rangle &= \sum_{n_k}\sum_{n_j}\sum_{m}(g_{n_k}^m g_{n_j}^m + h_{n_k}^m h_{n_j}^m)\left(\frac{R_E}{r}\right)^{(n_k+n_j+4)}\Bigg[\sin\theta_0 P_{n_j}^m(\cos\theta_0)\frac{dP_{n_k}^m(\cos\theta_0)}{d\theta} \\ &\quad + (n_k+n_j+1)(n_k+n_j+2)\frac{a}{2}\int\limits_{\theta=0}^{\theta_0} P_{n_k}^m(\cos\theta)P_{n_j}^m(\cos\theta)\sin\theta d\theta\Bigg] \qquad (15)\end{aligned}$$

The first term in the square bracket vanishes for $k = j$, $(k-m)$ odd, or $(j-m)$ even, as either $P_n^m(\cos\theta_0)$ or $dP_n^m(\cos\theta_0)/d\theta$ is zero due to the boundary conditions eqs (2) and (3). The elements of the damping matrix $\mathbf{\Lambda}$ in this case are:

$$f_{kj} = \left(1 - \frac{(n_k+n_j+2)}{(n_j-n_k)}\right)\left(\frac{R_E}{r}\right)^{n_k+n_j+4}\frac{a}{2}\sin\theta_0 P_{n_k}^m(\cos\theta_0)\frac{dP_{n_j}^m\cos\theta_0}{d\theta} \qquad (16)$$

for the non-diagonal elements with $m = m'$, $(k-m)$ even and $(j-m)$ odd,

$$f_{kk} = -(n_k+1)\left(\frac{R_E}{r}\right)^{2n_k+4} a\sin\theta_0 P_{n_k}^m(\cos\theta_0)\frac{\partial}{\partial n}\frac{dP_{n_j}^m(\cos\theta_0)}{d\theta} \qquad (17)$$

for the diagonal elements with $(k-m)$ even and

$$f_{kk} = (n_k+1)\left(\frac{R_E}{r}\right)^{2n_k+4} a\sin\theta_0\frac{dP_{n_j}^m(\cos\theta_0)}{d\theta}\frac{\partial}{\partial n}P_{n_k}^m(\cos\theta_0) \qquad (18)$$

for the diagonal elements with $(k-m)$ odd.

Another possibility is to minimise the 2 norm of the radial derivative of the field or its radial component, ensuring smoothness of the field with varying height. SCHA like SHA allows upward and downward continuation, and it is unreasonable to obtain a model that is smooth only on a particular surface, but becomes very rough a short distance above the Earth's surface. Such a behaviour would surely not represent the actual geomagnetic field. Deriving the expressions for both the $(dB_r/dr)^2$ and $(d\mathbf{B}/dr)^2$ norms is straightforward from the B_r^2 and $\mathbf{B}^2$ norms: the derivative only gives an additional factor of

$$(n_k+2)(n_j+2)\left(\frac{R_E}{r}\right)^2 \qquad (19)$$

Again all those equations are also valid for secular variation $\dot{\mathbf{B}}$ when substituting the time derivative of the coefficients $\{\dot{g}_k^m, \dot{h}_k^m\}$.

It is very important to test different norms and see how the change of damping factor for each of them influence the results.

2.3. Temporal modelling with splines

To model data as good as possible, simultaneously modelling in space and time is required. It can be done using penalised least squares splines. The overview of the spline method will be given in the next subsection.

Each of the SCHA Gauss coefficients can be expand in time on a basis of cubic B-splines $b_j(t)$

$$g_{n_k}^m(t) = \sum_{j=1}^{L} \alpha_{jkm} b_j(t) \tag{20}$$

The α_{jkm} are temporal coefficients to be determined. The following function might be minimise

$$(\gamma - \mathbf{Am})^T \mathbf{C}_e^{-1} (\gamma - \mathbf{Am}) + \int \left[\lambda \int \mathbf{B}^2 d\Omega + \tau \int \left(\frac{\partial^2 \mathbf{B}}{\partial t^2} \right)^2 d\Omega \right] dt \tag{21}$$

This three terms minimise the data misfit, spatial roughness and temporal roughness respectively. λ and τ are Lagrange multipliers controlling the trade off of the misfit and roughness criteria.

The individual epochs will be modelled separately as already has said and than compared with the results obtained by temporal modelling using splines. It will presumably show the advantage and better outcomes when modelling simultaneously in space and time.

2.3.1. Smoothing, Splines and Smoothing Splines

Practical problem that very ofen arises when analising geomagnetic data is how to fit a smooth curve of unknown parametric form to a time series of observations. One approach can be the use of cubic smoothing splines for estimating the unknown function. This means finding the smoothest twice continuously differentiable function which will fit the observations to a specific tolerance. Smoothing splines is very useful when the data are irregularly spaced and noisy. However, sometimes the method can become a hard comutational task. To avoid or reduce this problem, the least square splines can be used. In this approach the smoothness of the obtained function is control with the choice of the number of knots (the transition points t_j where the polynomials are joined) and the connection points for the spline basis functions which are piecewise cubic polynomials. For simple smoothing splines, the knots t_j are chosen to be the data points. If we have N observations y_i, i=1,...,N at points x_i from which we need to determine the function *f(x)* with the unknown parameters, we assume that:

$$y_i = f(x_i) + \epsilon_i \tag{22}$$

where $x_1 < x_2 <...< x_N$. ϵ_i are random, uncorrelated errors with zero mean and variance σ_i^2.

The misfit of data is given by:

$$\sum_{i=1}^{N}\left(\frac{y_i - f(x_i)}{\sigma_i}\right)^2 \tag{23}$$

The roughness is the square of the norm of the second derivative:

$$\int_{x_1}^{x_N}(\frac{\partial^2 f(x)}{\partial x^2})^2 dx \tag{24}$$

and it has to be minimized.

The solution of this problem is the minimizer $f_S(x)$ of the functional:

$$\sum_{i=1}^{N}\left(\frac{y_i - f(x_i)}{\sigma_i}\right)^2 + \lambda\int_{x_1}^{x_N}(\frac{\partial^2 f(x)}{\partial x^2})^2 dx \tag{25}$$

When having a large number of data, the comutational cost of finding the appropriate smooth curve may be high. For that reason the least square splines can be used. Then, the number of knots are reduced. The least square regression is used in the spline basic functionassociated with chosen knots. Of course than when appying the latest method, one has to keep in mind that choising the appropriate numbers of knots and their positions is a hard task. To overcome this disadvantage but retain the computational efficiency, the new term may be added in the square of the second derivative to the minimized functional. This modified form is well known as penalized least squares spline.

3. DATA MODELLING

3.1. Observatory data modelling

All the field component will be modelled, as well as their secular variations from the data set of 47 observatories for which it is expected to be of high quality and estimated data errors are of about 1 nT. The measurrements that I aquired span the period 1960-2002. With the physical regularization explained in the previous section the smoothes model will be found without forcing the smoothness by choosing the low truncation level, k_{max}. The compromise between the model smoothness and the misfit of the data will be considered, starting with the highest possible k_{max} allowed with the model. The trade-off curves for different norms will be plotted and based on them, the most appropriate damping factor has to be choosen. The same will be done when modelling individual epoch as well as with applying temporal model procedures.

3.2. Synthetic data modelling: Comprehensive model (CM3)

Traditionally, fields from different sources have been modelled separatelly. When modelling the core SV at the core-mantle boundary from observatory first difference, survay and satellite data, Bloxham & Jackson (1992), Jackson et al. (2000) included the effects of crustal and ionospheric field as noise sources, but neglected field external to satellite sampling regions. The problem is that this excluded external field can be coestimated with the internal fields. Spherical harmonic models of ionospheric field have usually been produced separately from other fields. Global model of the lithospheric field are obtained by removing estimates of the main, ionospheric and magnetospheric fields from the data. The other possibility is to include harmonics of higher degree when expanding the potential of the internal field. In all of this attempts, it appears the effect of frequency overlap between the spectra of the various field sources in spatial as well as in temporal domains. The careful study (Sabaka et al., 2002) showed that joint analysis of surface and satellite data can give the possibility to resolve magnetospheric, ionospheric and internal sources. The approach for simultaneous inversion of all sources is called ”comprehensive approach”. The model has been derived from quiet-time Magsat and POGO satellite and observatory hourly means measurements for the period 1960-1985. It takes into the account the main field influences on the magnetosphere, main field and solar influences on the coupling currents, influence of the ionosphere and the magnetosphere on Eart-induced fields and explicit estimation of the litospheric field. The model describes well 591 432 data with 16 594 parameters, what results in data-to-parameter ratio of 36, what is larger than in the case of other field models in used.

Using the comprehensive model (CM3), the synthetic data will be produce for the same region as covered with the observatories. It is good to compare the time series of the synthetic data with those for the real data. If the trends are similar, but curves just shifted it is most likely that the considered observatory is on a magnetic anomaly what cannot be detected by global model. The synthetic data should then be modelled in order to test the used computer code and to give the idea how smooth the field we can expect. Comparing the spatial power spectra of the SV field on different spherical caps and of the global model is good approach in selecting the appropriate cap angle. It should be looked for the best model behaviour at lower harmonics (it is expected that the most important harmonic contribution comes from the first harmonic). Furthermore, the synthetic data are needed to feel the regions without real data near the cap boundaries in order to avoid artifical edge effects caused by numerical instability.

4. CONCLUSION

By using the good time-series of the data from geomagnetic observatories which are dense enough in the Europian region,the appropriate regional model can be derived. The adventage of all techniques described in previous sections are:

- using physical regularization

- choice of the spherical cap angle based on power spectra of the field components and SV field
- using Comprehensive model as a reference model instead of others global models which are avaleable
- comparison with the CHAMP satellite data model

  The regional model which will be developed is aimed to reveal possible lithospheric secular variation anomalies and throw light on the problem of the existence of the medium-scale to small-scale structure in the SV data. Maps will be produced using GMT, Matlab and Surfer packages.

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